4. V. N. Nikolaevskii, "Dynamic strength and fracture rate," in: Collection. Mechanics [in Russian], No. 26 (1981).
5. L. A. Galin and G. P. Cherepanov, "Self-sustaining fracture of a stressed brittle body," Dokl. Akad. Nauk SSSR, 167, No. 3 (1966).
6. S. S. Grigoryan, "Some problems of the mathematical theory of the deformation and fracture of rocks," Prikl. Mat. Mekh., 31, No. 4 (1967).
7. L. I. Slepyan, "On the brittle fracture wave," Inzh. Zh. Mekh. Tverd. Tela, No. 4 (1968).
8. L. I. Slepyan and L. V. Troyankina, "Fracture of a brittle bar in longitudinal impact," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 2 (1969).
9. L. I. Slepyan, "Dynamics of a crack in a grid," Dokl. Akad. Nauk SSSR, 258, No. 3 (1981).
10. L. I. Slepyan, "Propagation of a crack in a rapidly vibrating grid," Dokl. Akad. Nauk SSSR, 260, No. 3 (1981).
11. L. I. Slepyan, "Antiplane problem of a crack in a grid," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 5 (1982).
12. L. I. Slepyan, "On the relation between the solutions of the mixed dynamic problems for an elastic continuum and a grid," Dokl. Akad. Nauk SSSR, 266, No. 3 (1982).

STATISTICAL ESTIMATE OF BRITTLE STRENGTH WITH ALLOWANCE FOR CRACK RESISTANCE

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UDC 620.171.3

The classical statistical approach to the question of the probability of brittle fracture presupposes the presence in the investigated material of a system of defects, which also determines the strength of a specimen made of the material concerned. It is assumed that for each specific defect there is a corresponding local strength. The strength of the specimen as a whole (at least under static loading) is determined by the strength of the most dangerous defect, which in a given specimen has the minimum strength. The scale effect (SE) consists in the fact that in a specimen of greater volume there is a greater probability of encountering a more dangerous defect. Such an explanation of the SE was first given in [1], and a mathematical treatment using several different approaches was presented first in [2] and later in [3].

In [2] Weibull introduced the concept of the probability of fracture S_0 of unit volume at the stress σ, and on the basis of a solution of the statistical problem obtained the fracture probability S at stress σ for a specimen of volume V:

$$S = 1 - e^{-Vn(\sigma)}, \tag{1}$$

where the function $n(\sigma)$ is taken in the form

$$n(\sigma) = (\sigma/\sigma_0)^m \tag{2}$$

(σ_0 and m are experimentally selected material constants). Then, in [2] from Eq. (1), using (2), the following relation between the breaking stress and the volume of the test specimen was obtained:

$$\sigma_p = \sigma_0 I_m V^{-1/m},$$

where I_m is a constant for a given state of stress. In more general form

$$\sigma_p = A V^{-1/m}, \tag{3}$$

where $A = \sigma_0 I_m$.

Another approach to the solution of the problem is proposed in [3], namely to find the probability W(F)dF that in a specimen of volume V the most dangerous defect is that with the parameter $F \div F + \Delta F$

$$W(F)\,dF = \bar{n} V p(F) \left[\int_F^\infty p(F)\,dF\right]^{\bar{n}V-1} dF,$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 135-138, November-December, 1984. Original article submitted August 5, 1983.

where $\bar{n}$ is the average number of defects per unit volume; p(F) is the probability density function of the defect parameter. Here, by defect parameter we understand the value of the brittle strength. In this case the most probable value of the brittle strength F* of specimens of a given volume V is determined from the maximum condition $\partial W(F)/\partial F = 0$.

Using the Gaussian distribution for p(F), the authors of [3] obtained the following equations for the breaking stress as a function of the volume of the test specimen:

$$F^* = a + b/V,\ F^* = F_0 - \sqrt{A \operatorname{tg} V + B}$$

for specimens of "small" and "large" volume respectively. Here, a, b, F_0, A, and B are constants.

Below we propose a solution of the statistical problem of the probability of brittle fracture that relies on linear fracture mechanics and the concept of the weakest link [4]. Let a brittle material possess a certain system of defects in the form of elliptical Griffiths cracks. Then, other things being equal, the strength of a specimen made of the material in question will be determined by the crack with the maximum size.

In order to solve the problem we will employ the approach proposed in [3]; however, as the defect parameter we will take not the brittle strength but a function associated with the characteristic dimension of the crack.

Let $F(\xi)$ be the probability density function of the defect parameter. We assume that the parameter ξ, without being physically defined, is an increasing function $\xi(a)$, where a is the characteristic crack dimension. We also assume that the function $F(\xi)$ has a region of definition from ξ_1 to ξ_2. The probability that a crack taken at random has the parameter ξ^* is equal to $F(\xi^*)d\xi$. In this case the probability $\int_{\xi_1}^{\xi^*} F(\xi)d\xi$, corresponding to the presence of a crack with parameter ξ^* in a specimen of volume V (all the other cracks having a parameter less than ξ^*) is equal to

$$P(\xi^*)\,d\xi = \bar{n}VF(\xi^*)\left[\int_{\xi_1}^{\xi^*} F(\xi)\,d\xi\right]^{\bar{n}V-1} d\xi,$$

where

$$P(\xi) = \bar{n}VF(\xi)\left[\int_{\xi_1}^{\xi} F(\xi)\,d\xi\right]^{\bar{n}V-1} \tag{4}$$

is the corresponding probability density function; $\bar{n}$ is the average number of cracks per unit volume. The most probable (modal) value of the maximum of the defect parameter for a specimen of given volume is found from the expression

$$\partial P(\xi)/\partial\xi = 0. \tag{5}$$

Differentiating Eq. (4) with allowance for (5), we obtain

$$F(\xi)^2(\bar{n}V - 1) + \int_{\xi_1}^{\xi} F(\xi)\,d\xi\,\frac{\partial F(\xi)}{\partial\xi} = 0. \tag{6}$$

Let $F(\xi)$ have a normal distribution, i.e.,

$$F(\xi) = \frac{1}{S\sqrt{2\pi}}\exp\left[-\frac{(\xi - m)^2}{2S^2}\right],$$

where S and m are the distribution parameters. In this case

$$\xi_1 = -\infty,\ \frac{\partial F(\xi)}{\partial\xi} = -F(\xi)\frac{\xi - m}{S^2},\ \int_{-\infty}^{\xi} F(\xi)\,d\xi = \Phi^*\left(\frac{\xi - m}{S}\right),$$

where $\Phi^*((\xi - m)/S)$ is the error function. Taking the above into account, we reduce Eq. (6) to the form

$$\frac{\bar{n}V-1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\xi-m}{S}\right)^2\right]-\left(\frac{\xi-m}{S}\right)\Phi^*\left(\frac{\xi-m}{S}\right)=0. \tag{7}$$

In order to solve the equation obtained, we carry out the following transformations. We introduce the new variables

$$N=\bar{n}V, \tag{8}$$

where N is the total number of cracks in the test volume;

$$t=(\xi-m)/S. \tag{9}$$

In this case Eq. (7) becomes

$$N=\sqrt{2\pi}t\exp(t^2/2)\Phi^*(t)+1. \tag{10}$$

Using the equation obtained, we can plot the function t = t(N) (Fig. 1) or obtain it in tabular form. Then, using (8)-(10), we obtain the following relation between the most probable value of the maximum defect parameter and the volume of the test specimen:

$$\xi=t(\bar{n}V)S+m. \tag{11}$$

The value of ξ obtained is a modal value corresponding to a quantile of order P of distribution (4).

In this case the quantile of order P of the breaking stress for the corresponding distribution is found from the expression

$$\sigma_b=f(a(\xi_P)), \tag{12}$$

where f is a function of the relation between the breaking stress and the characteristic crack dimension; $a(\xi)$ is the inverse function of $\xi(a)$.

We now turn to the determination of the breaking stress as a function of test specimen volume, assuming as a first approximation that σ_b corresponds to the average value of the breaking stress. We will consider an infinite solid with an internal discoidal crack of diameter a. In the case of a tensile stress σ we have [5]

$$K_I=\sigma\sqrt{2a/\pi},$$

where K_I is the stress intensity coefficient. Since in the case considered the crack diameter is much less than the characteristic dimensions of the body, it is possible to assume that the body is infinite; then the expression for the breaking stress takes the form

$$\sigma_b=K_{IC}\sqrt{\pi/2a}, \tag{13}$$

where K_{IC} is the critical stress intensity coefficient. In Eq. (13), as a we will take the value a_P; then, in accordance with Eqs. (11)-(13), taking into account the above-mentioned assumption, we obtain the following relation between the strength of the specimen and the test volume:

$$\sigma_b=K_{IC}\sqrt{\pi/[2a(t(\bar{n}V)S+m)]}. \tag{14}$$

It is convenient to consider the form of the function $\xi(a)$ with reference to the microstructure of a foam plastic; in this case we treat the pores as defects and take the pore diameter as the characteristic dimension. This material has the advantage that even at low magnification it is possible to obtain a quite accurate estimate of the volume pore density and the pore sizes [6]. In Fig. 2 we have constructed a pore diameter histogram for PPU-307 foam plastic (γ = 6.9 kN/mm$^{3/2}$ which is closely approximated by a logarithmically normal distribution. In this case Eq. (14) can be written in the form

$$\sigma_b=K_{IC}\sqrt{\pi/[2\exp(t(\bar{n}V)S+m)]}. \tag{15}$$

Table 1 gives the experimental results of tensile tests on PPU-307 foam plastic (γ = 6.9 kN/m^3) and the approximate results obtained using Eq. (15); in this case the values $\bar{n}$ = 206 per mm^3, S = 0.524, and m = −2.082 were obtained from an analysis of the microstructure, and K_{IC} = 16.5 N/mm$^{3/2}$ from the results of the approximation. The tabulated data indicate good agreement between experiment and calculation and confirm the reliability of the equation obtained. The value K_{IC} = 16.5 N/mm$^{3/2}$ obtained as a result of the approximation is of the same order as the values of K_{IC} for nonmetals [7].

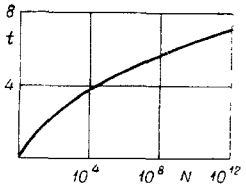

Fig. 1

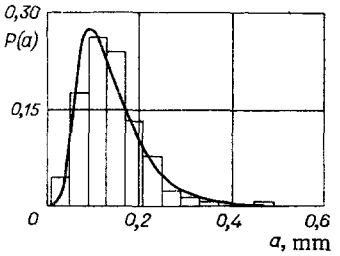

Fig. 2

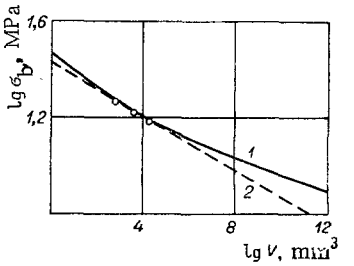

Fig. 3

TABLE 1

V, mm^3	N	t	σ_b, MPa (experiment)	σ_b, MPa (calculation)
$8{,}04\cdot10^2$	$1{,}74\cdot10^5$	4,40	18,6	18,5
$4{,}31\cdot10^3$	$9{,}31\cdot10^5$	4,75	16,9	16,9
$2{,}17\cdot10^4$	$4{,}69\cdot10^6$	5,06	15,5	15,6

As a second example it is possible to take a sintered material with artificially organized defects, for example VNMZ-2.* Thus, for VNMZ-2 the experimental values of the strength were 536 and 455 MPa and the calculated values 547 and 445 MPa for volumes of $2.83\cdot10^1$ and $4.71\cdot10^3$ mm^3 respectively.

In order to describe arbitrary scale effect results without a preliminary analysis of the microstructure, it is desirable to reduce Eq. (15) to the form

$$\sigma_b = A \exp\left[-Bt(\overline{nV})\right], \tag{16}$$

where $A = K_{IC}\sqrt{\pi/[2\exp(m)]}$; $B = S/2$. The equation obtained makes it possible to describe the experimental results over a broad range of variation of the test volume. In this respect Eq. (16) is more convenient than the scale effect equations obtained in [3]. Thus, in [3], owing to a number of functional approximations, two equations were used: one for "small-volume" and the other for "large-volume" specimens. Such differentiation is inconvenient and prevents extrapolation.

It is interesting to compare Eq. (3) with our Eq. (16). With Eq. (3) it is possible to approximate the experimental results for any part of the scale dependence but, as shown in [8], it cannot be used to describe the results where there is a considerable variation in the volume of the test specimens. In [8], to overcome this disadvantage, it is proposed to add a constant term on the right of Eq. (3), which does not fit in with the theoretical premises.

In Fig. 3 we have plotted σ_b against the specimen volume for PPU-307 foam plastic (γ = 6.9 kN/m^3). These results were approximated by means of both Eq. (3) (curve 2) and Eq. (16) (curve 1). It is clear from Fig. 3 that in the case of Eq. (16) there is a diminution in the intensity of the scale effect $[\partial(\log \sigma_b)/\partial(\log V)]$ with increase in test volume, as observed in practice [8]. In the case of Eq. (3), however, the intensity of the scale effect remains constant and equal to 1/m. Curves 1 and 2 (see Fig. 3) coincide over an internal variation of the volume of approximately three orders. Hence it follows that in describing the experimental results of the scale dependence, for a small change in volume it is possible to use Eq. (3); however, extrapolation is then excluded. Our Eq. (15) (or (16)) makes it possible to approximate the experimental results over a much greater range of variation of the test volume and permits long-range extrapolation.

LITERATURE CITED

1. A. P. Aleksandrov and S. N. Zhurkov, The Brittle Fracture Effect [in Russian], Gostekhteoretizdat, Moscow–Leningrad (1933).
2. W. Weibull, "A statistical theory of the strength of materials," Roy. Swedish Inst. Engng. Res. Proc., No. 151 (1939).

*VNMZ-2 alloy is obtained by hot pressing from W, Ni, and Cu powders and accordingly, as with foam plastic, it is possible to estimate the number and distribution of the defects. The material contains 3% Ni, 2% Cu, and 95% W by weight.

3. T. A. Kontorova and O. A. Timoshenko, "Extension of the statistical theory of strength to the case of an inhomogeneous state of stress," Zh. Tekh. Fiz., 19, No. 3 (1949).
4. A. M. Freudenthal, "Statistical approach to brittle fracture," in: Fracture. Mathematical Basis of the Theory of Fracture, Vol. 2 [Russian translation], Mir, Moscow (1975).
5. D. Broek, Fundamentals of Fracture Mechanics [in Russian], Vyssh. Shkola, Moscow (1980).
6. V. A. Pavlov, Foam Polystyrene [in Russian], Khimiya, Moscow (1973).
7. G. P. Cherepanov, Mechanics of Brittle Fracture [in Russian], Nauka, Moscow (1974).
8. A. V. Matveev, N. I. Shleifer, and V. I. Konstantinov, "Effect of test specimen dimensions on the strength of K8 glass," Opt.-Mekh. Promyshlennost', No. 2 (1979).

TEMPERATURE–FREQUENCY DEPENDENCE OF MECHANICAL LOSSES UNDER PERIODIC DEFORMATION OF LAMINAR GLASS-CARBON PLASTICS

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UDC 534.28+532.135

Experimental results [1-3] indicate that under periodic deformation the temperature–frequency locations of the relaxation maximums of the tangent of the mechanical loss angle $\tan\delta$ of a laminar composite and the material of its matrix do not agree. The reasons and regularities for such a shift of the $\tan\delta$ maximums remain unexplained within the framework of these papers. Meanwhile it is shown theoretically that the insertion of an elastic filler in a polymer material as well as the passage from shear over to longitudinal or bending vibrations in unfilled polymers and composites will distort the relaxation spectrum and change the effective relaxation time [4].

Regularities in the temperature–frequency location and the magnitude of the loss tangent maximum and the real part of the Young's modulus in laminar composites are examined below. The mixture rules proposed in [5, 6] were used here. The formulas in [6] are approximate and convenient for utilization for a large number of constituents in the composite. Moreover, they permit easy evaluation of the stress concentration coefficients in the composite material [7]. As is mentioned in [8], such approaches that take account of the actual mode of interaction between the composite constituents will permit obtaining results that are in satisfactory agreement with test data and are consequently adequate for technical applications.

1. Let a composite, which is transversally isotropic on the average, consist of a viscoelastic matrix and an elastic filler. The stochastic inhomogeneity of the composite is not taken into account. The energy dissipation mechanism is related only to the inelastic behavior of the matrix [9]. There is no relaxation of the bulk modulus K in a viscoelastic composite. Shear relaxation is described by the Yu. N. Rabotnov kernel, i.e., in the operator representation the shear modulus of the viscoelastic component is written as follows [10]:

$$\hat{G} = G_\infty[1 - \chi\hat{\vartheta}(x)], \tag{1.1}$$

where $\hat{\vartheta}(x)$ is the Yu. N. Rabotnov resolvent operator while the rheological parameters x, χ are expressed in terms of the unrelaxed G_∞ and relaxed G_0 values of the shear modulus and the effective relaxation time τ_ε:

$$x = -\tau_\varepsilon^{-\gamma}, \quad \chi = (G_\infty - G_0)/G_\infty\tau_\varepsilon^\gamma \quad (0 < \gamma \leqslant 1), \tag{1.2}$$

where γ is the kernel singularity parameter.

The expression for the Young's modulus operator of the viscoelastic constituent has the form [4]

$$\hat{E} = E_\infty\left[1 - \chi_E\hat{\vartheta}(x_E)\right],$$
$$x_E = -\tau_E^{-\gamma} = x + G_\infty\chi/(3K + G_\infty), \quad E_\infty = 9KG_\infty/(3K + G_\infty), \tag{1.3}$$
$$\chi_E = \eta\chi, \; 1/\eta = 1 + G_\infty/3K.$$

Kuibyshev. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 139-144, November-December, 1984. Original article submitted October 14, 1983.

0021-8944/84/2506- 0931$08.50